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PALATINI $f(R)$ COSMOLOGY

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We investigate the modified gravity theories in terms of the effective dark energy models. We compare the cosmic expansion history and the linear growth in different models. We also study the evolution of linear cosmological perturbations in modified theories of gravity assuming the Palatini formalism. We find the stability of the superhorizon metric evolution depends on models. We also study the matter density fluctuation in the general gauge and show the differential equations in super and sub-horizon scales.

Keywords: Modified Gravity; Palatini Formalism.

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1. Introduction

Type Ia supernova distance-redshift measurement shows that the present expansion of the universe is accelerating ¹. To explain this phenomena we need to introduce a homogeneous component of energy with a negative pressure, dubbed dark energy ², or a modification of gravity ^{3,4,5,6}.

We can parameterize the effective dark energy component in the modified gravity theories ⁷. The evolution equation of Hubble parameter and that of energy density fluctuation can be simply expressed with this parametrization.

The Palatini formalism where the metric and the connections are treated as independent variables and the energy momentum tensor only depends on the metric leads to a different theory from what is obtained from the metric formalism. The Palatini formalism of $f(R)$ gravity results in second order differential equations due to the algebraic relation between the curvature scalar and the trace of the energy momentum tensor. We will use the evolution of linear perturbations in $f(R)$ models in the physical frame to check the stability of the theory. We also investigate the evolution of the matter density fluctuation.

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In the next section we introduce the parametrization of modified gravities as an effective dark energy. We review the Palatini $f(R)$ gravity in section 3. In section 4, we investigate the linear perturbation of $f(R)$ gravity. We also derive the stability equation of metric fluctuations in the high curvature limit and show the stability in a specific model. We show the evolutions of the metric fluctuation and the density fluctuation in the superhorizon and the subhorizon scales in section 5. We reach our conclusions in section 6.

2. Modified Gravity as An Effective Dark Energy

In general, the modified gravity can be expressed in the following form

$$H^2 - \delta H = \frac{8\pi G}{3} \rho_m, \quad (1)$$

where δH represents the modification to the Friedmann equation of general relativity and ρ_m is the matter density. We can rewrite this equation as the effective dark energy term

$$H^2 = \frac{8\pi G}{3} \left(\rho_m + \rho_{\text{DE}} \right), \quad (2)$$

where we define the effective dark energy density as $8\pi G \rho_{\text{DE}} = 3\delta H$. From this equation (2) we can find the effective equation of state of dark energy

$$\omega_{\text{DE}} = -1 - \frac{1}{3} \frac{d \ln \delta H}{d \ln a}. \quad (3)$$

We can also express the expansion history of the universe in very simple way for the specific case. If δH is proportional to H (i.e. $\delta H = \alpha H$), then we can find the Hubble parameter $H(z)$

$$\frac{H}{H_0} = \frac{1}{2} \left(\frac{\alpha}{H_0} + \sqrt{\left(\frac{\alpha}{H_0} \right)^2 + 4\Omega_m^{(0)}(1+z)^3} \right). \quad (4)$$

Table 1. Comparison of δH and ω_{DE} in BD, DGP, metric formalism $f(R)$ theory, and Palatini formalism $f(R)$ models. $F(R) = \frac{\partial f(R)}{\partial R}$ and F_0 is the present value of $F(R)$.

	δH	ω_{DE}
BD	$\frac{\omega_{\text{BD}}}{6} \frac{\dot{\phi}^2}{\phi^2} - H \frac{\dot{\phi}}{\phi}$	$-1 + \frac{\frac{\ddot{\phi}}{\phi} - H \frac{\dot{\phi}}{\phi} + \omega_{\text{BD}} \frac{\dot{\phi}^2}{\phi^2}}{\frac{\omega_{\text{BD}}}{2} \frac{\dot{\phi}^2}{\phi^2} - 3H \frac{\dot{\phi}}{\phi}}$
DGP	$\frac{H}{r_0}$	$-\frac{1}{1+\Omega_m}$
Metric $f(R)$	$\frac{1}{3F_0} \left(\frac{1}{2}(FR - f) - 3H\dot{F} + 3H^2(F_0 - F) \right)$	$-1 + \frac{2\ddot{F} - 2H\dot{F} - 4\dot{H}(F_0 - F)}{FR - f - 6H\dot{F} + 6H^2(F_0 - F)}$
Palatini $f(R)$	$\frac{1}{3F_0} \left(\frac{1}{2}(FR - f) + \frac{3}{2}\ddot{F} + \frac{3}{2}H\dot{F} - \frac{3}{2}F\frac{\ddot{F}^2}{F^2} + 3H^2(F_0 - F) \right)$	$-1 + \frac{2\ddot{F} - 2H\dot{F} - 3F\frac{\ddot{F}^2}{F^2} - 4\dot{H}(F_0 - F)}{FR - f + 3\ddot{F} + 3H\dot{F} - 3F\frac{\ddot{F}^2}{F^2} + 6H^2(F_0 - F)}$

In table 1, we show the expressions of the modification term to the Friedmann equation (δH) and the effective equation of state of dark energy (ω_{DE}) for the

different modified gravity models. BD represents Brans-Dicke theory, DGP stands for the brane model of Dvali, Gabadadze, and Porrati⁴, metric $f(R)$ means the $f(R)$ theory in the metric formalism⁵, and Palatini $f(R)$ does that in the Palatini formalism⁶. In the Palatini formalism we can relate the functions $f(R)$ and $F(R)$ to the matter energy density ρ_m .

Table 2. Comparisons of $H(z)$ and evolution of the linear perturbation δ_m with the effective gravitational constant G_{eff} in each model. In metric and Palatini formalism the perturbation calculations are held for subhorizon scale (i.e. $\frac{k}{a} > H$). Where $Q = -\frac{2F_{,R}}{F} \frac{k^2}{a^2}$ and $F'(T) = \frac{\partial F(T)}{\partial T}$.

	$\frac{H(z)}{H_0}$	δ_m	G_{eff}
BD	$\sqrt{\frac{\delta H}{H_0^2} + \frac{\phi_0}{\phi} \Omega_m^{(0)} (1+z)^3}$	$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta = 0$	$\left(\frac{2\omega_{\text{BD}} + 4}{2\omega_{\text{BD}} + 3} \right) \frac{1}{\phi_0}$
DGP	$\frac{1}{2} \left(\frac{1}{r_0 H_0} + \sqrt{\left(\frac{1}{r_0 H_0} \right)^2 + 4\Omega_m^{(0)} (1+z)^3} \right)$	$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta = 0$	$G \left(1 + \frac{1}{3[1+2r_0 H \omega_{\text{DE}}]} \right)$
Metric $f(R)$	$\sqrt{\frac{\delta H}{H_0^2} + \Omega_m^{(0)} (1+z)^3}$	$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta = 0$	$\frac{2(1-2Q)}{2-3Q} \frac{G}{F}$
Palatini $f(R)$	$\sqrt{\frac{\delta H}{H_0^2} + \Omega_m^{(0)} (1+z)^3}$	$\ddot{\delta} + H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta = 0$	$-\frac{1}{4\pi} \frac{F'}{2F+3F'} \frac{k^2}{a^2}$

We show the expansion rate $H(z)/H_0$ where H_0 is the present value of the Hubble parameter, the evolution equation for the energy density perturbation δ_m , and the effective gravitational constant G_{eff} for different models in table 2. The evolution equations of δ_m for the $f(R)$ models are suitable for the subhorizon scale. The scale dependence on G_{eff} is the one of the major differences compared to the general relativity.

3. Palatini $f(R)$ Gravity

We consider a modification to the Einstein-Hilbert action assuming the Palatini formalism, where the metric $g_{\mu\nu}$ and the torsionless connection $\hat{\Gamma}_{\mu\nu}^\alpha$ are independent quantities and the matter action depends only on metric

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} f(\hat{R}(g_{\mu\nu}, \hat{\Gamma}_{\mu\nu}^\alpha)) + \mathcal{L}_m(g_{\mu\nu}, \psi) \right], \quad (5)$$

where ψ are matter fields. Then the Ricci tensor is defined solely by the connection

$$\hat{R}_{\mu\nu} = \hat{\Gamma}_{\mu\nu, \alpha}^\alpha - \hat{\Gamma}_{\mu\alpha, \nu}^\alpha + \hat{\Gamma}_{\alpha\beta}^\alpha \hat{\Gamma}_{\mu\nu}^\beta - \hat{\Gamma}_{\mu\beta}^\alpha \hat{\Gamma}_{\alpha\nu}^\beta, \quad (6)$$

whereas the scalar curvature is given by $\hat{R} = g^{\mu\nu} \hat{R}_{\mu\nu}$. Then we can find the Ricci tensor and the scalar curvature by using the metric relation

$$\hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2} \frac{1}{F^2} \nabla_\mu F \nabla_\nu F - \frac{1}{F} \nabla_\mu \nabla_\nu F - \frac{1}{2} g_{\mu\nu} \frac{1}{F} \square F, \quad (7)$$

$$\hat{R} = R - 3 \frac{1}{F} \square F + \frac{3}{2} \frac{1}{F^2} (\partial F)^2. \quad (8)$$

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We can derive the field equation of $f(R)$ gravity in the Palatini formalism from the above action (5)

$$F(\hat{R})\hat{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(\hat{R}) = 8\pi GT_{\mu\nu}, \quad (9)$$

where $F(\hat{R}) = \partial f(\hat{R})/\partial \hat{R}$ and the matter energy momentum tensor is given as usual form $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$. From the above equations we can get the generalized Einstein equation

$$G_{\mu\nu} = 8\pi GT_{\mu\nu} + (1-F)R_{\mu\nu} - \frac{3}{2}\frac{1}{F}\nabla_\mu F\nabla_\nu F + \nabla_\mu\nabla_\nu F + \frac{1}{2}(f-R)g_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\square F. \quad (10)$$

From the above equation we can derive the modified Friedmann equations

$$3H^2 = \frac{8\pi G}{2F}\rho + \frac{f}{2F} - 3H\frac{\dot{F}}{F} - \frac{3}{4}\frac{\dot{F}^2}{F^2}, \quad (11)$$

$$-2\dot{H} = \frac{8\pi G}{F}(\rho + p) + \frac{\ddot{F}}{F} - H\frac{\dot{F}}{F} - \frac{3}{2}\frac{\dot{F}^2}{F^2}, \quad (12)$$

where dot denote derivatives with respect to t .

4. Linear Perturbation and Stability in Palatini $f(R)$ Gravity

The line element in the conformal Newtonian gauge is given by

$$ds^2 = a^2(\tau) \left[-\left(1 + 2\Psi(\tau, \vec{x})\right)d\tau^2 + \left(1 - 2\Phi(\tau, \vec{x})\right)dx^i dx_i \right]. \quad (13)$$

The main modifications for viable models with stable high curvature limits happen well during the matter dominated epoch and we can take the components of the energy momentum tensor as

$$T_0^0 = -\rho(1 + \delta), \quad T_i^0 = \rho\partial_i q, \quad T_j^i = 0. \quad (14)$$

From the equation (10), we can find the perturbed Einstein equation

$$\begin{aligned} F\delta G_\nu^\mu &= \kappa^2\delta T_\nu^\mu - R_\nu^\mu\delta F - \frac{3}{2}\frac{\delta(\nabla^\mu F\nabla_\nu F)}{F} + \frac{3}{2}\frac{\nabla^\mu F\nabla_\nu F}{F^2}\delta F + \delta(\nabla^\mu\nabla_\nu F) \\ &+ \left(\frac{3}{2}\frac{\square F}{F}\delta F + \frac{3}{4}\frac{\delta(\partial F)^2}{F} - \frac{3}{2}\frac{(\partial F)^2}{F^2}\delta F - \delta\square F\right)\delta_\nu^\mu, \end{aligned} \quad (15)$$

where we use $\delta f(\hat{R}) = F(\hat{R})\delta\hat{R}$. If we consider the ij -component of the perturbed equation, then we can find

$$\Phi - \Psi = \frac{\delta F}{F}, \quad (16)$$

where we assume the null anisotropic stress. If we use the above equation (16), then we can express the other components of the perturbed Einstein equation (15)

$$3H^2 \left[\Phi' + \Psi' + \frac{1}{2} \frac{F'}{F} (\Phi' + \Psi') + \left(\frac{1}{2} \frac{F''}{F} - \frac{1}{2} \frac{F'^2}{F^2} + \frac{1}{2} \frac{H'}{H} \frac{F'}{F} + \frac{1}{2} \frac{F'}{F} + \frac{H'}{H} + 1 \right) \Phi + \left(-\frac{1}{2} \frac{F''}{F} + \frac{F'^2}{F^2} - \frac{1}{2} \frac{H'}{H} \frac{F'}{F} + \frac{3}{2} \frac{F'}{F} - \frac{H'}{H} + 1 \right) \Psi \right] + \frac{k^2}{a^2} (\Phi + \Psi) = -\frac{\kappa^2 \rho}{F} \delta, \quad (17)$$

$$H \left[\Phi' + \Psi' + \Phi + \Psi + \frac{1}{2} \frac{F'}{F} (\Phi + \Psi) \right] = -\frac{\kappa^2 \rho}{F} q, \quad (18)$$

$$3H^2 \left[\Phi'' + \Psi'' + \left(4 + \frac{H'}{H} \right) \Phi' + \left(3 \frac{F'}{F} + 4 + \frac{H'}{H} \right) \Psi' + \left(-\frac{F''}{F} - \left(2 + \frac{H'}{H} \right) \frac{F'}{F} + \frac{H'}{H} + 3 \right) \Phi + \left(3 \frac{F''}{F} + \left(6 + 3 \frac{H'}{H} \right) \frac{F'}{F} + 3 \frac{H'}{H} + 3 \right) \Psi \right] = 0. \quad (19)$$

where primes denote derivatives with respect to $\ln a$. To capture the metric evolution, let us introduce two parameters as in the reference⁸: θ the deviation from ζ conservation and ϵ the deviation from the superhorizon metric evolution

$$\zeta' = \Phi' + \Psi - H' q = -\frac{H'}{H} \left(\frac{k}{aH} \right)^2 B \theta, \quad (20)$$

$$\Phi'' + \Psi' - \frac{H''}{H'} \Phi' + \left(\frac{H'}{H} - \frac{H''}{H'} \right) \Psi = -\left(\frac{k}{aH} \right)^2 B \epsilon, \quad (21)$$

where we define the dimensionless quantity

$$B = \frac{F'}{F} \frac{H}{H'}. \quad (22)$$

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From the above equations, we can find the expression for θ and ϵ ,

$$\begin{aligned} \frac{H'}{H} \left(\frac{k}{aH} \right)^2 B\theta = \frac{1}{2} \left[\frac{B'}{B} + \frac{3}{2} \frac{H'}{H} B + \frac{H''}{H'} + 4 \right] (\Phi - \Psi) + \frac{1}{2} \left[-\frac{H'}{H} B' + \left(\frac{H'}{H} - \frac{H''}{H} \right) B \right. \\ \left. + \frac{1}{2} \frac{H'^2}{H^2} B^2 \right] Hq \end{aligned} \quad (23)$$

$$\begin{aligned} \left(\frac{k}{aH} \right)^2 B\epsilon = -\frac{1}{2} \left[2 \frac{B'^2}{B^2} + \left(5 \frac{H''}{H'} + \frac{H'}{H} + 9 \right) \frac{B'}{B} + \frac{H'}{H} B' + \frac{H'^2}{H^2} B^2 + \left(\frac{5}{2} \frac{H''}{H} + \frac{3}{2} \frac{H'^2}{H^2} \right. \right. \\ \left. \left. + 6 \frac{H'}{H} \right) B + \frac{H''}{H} + 3 \frac{H''^2}{H'^2} + 13 \frac{H''}{H'} + \frac{H'}{H} + 9 + 2 \left(\frac{H''}{H'} + \frac{H'}{H} + 3 \right) \frac{1}{B} \right] \\ (\Phi - \Psi) + \left[\frac{H'}{H} B' + \frac{1}{2} \frac{H'^2}{H^2} B^2 + \left(\frac{H''}{H} + \frac{3}{2} \frac{H'}{H} \right) B \right] \Psi - \frac{1}{2} \left[\frac{H'}{H} B + \frac{H''}{H'} \right. \\ \left. + \frac{H'}{H} + 3 \right] \frac{\kappa^2 \rho}{FH} q \end{aligned} \quad (24)$$

From equation (23), we can recover the conservation of Newtonian gauge when $\Phi = \Psi$ and F is a constant.

In addition to these equations, we can find very useful equation from the structure equation (9)

$$\frac{\delta F}{F} = -\frac{1}{3} \frac{F'}{F} \delta = \Phi - \Psi. \quad (25)$$

From this equation we can find the evolution equation of matter density fluctuation

$$\begin{aligned} \delta'' + \left(2 \frac{B'}{B} + \frac{H'}{H} B + 2 \frac{H''}{H'} - \frac{H'}{H} + 3 \right) \delta' + \left(\frac{H'}{H} B' - 2 \frac{B'^2}{B^2} - \left[4 \frac{H''}{H'} + 2 \frac{H'}{H} + 4 \right] \frac{B'}{B} \right. \\ \left. + \frac{H'^2}{H^2} B^2 + \left[\frac{H''}{H} - \frac{H'^2}{H^2} + 5 \frac{H'}{H} \right] B + \left[-3 \frac{H''}{H} - 4 \frac{H''}{H'} - 2 \frac{H''^2}{H'^2} + \frac{H'^2}{H^2} - 2 \frac{H'}{H} + 6 \right] \right. \\ \left. - 4 \left[\frac{H''}{H'} + \frac{H'}{H} + 3 \right] \frac{1}{B} \right) \delta = -3\Psi'. \end{aligned} \quad (26)$$

Compared with previous works⁹, we do not specify the gauge of matter density to solve the matter density fluctuation.

Unstable metric fluctuations can create order unity effects that invalidate the background expansion history. We can derive the evolution equation of the deviation parameter¹⁰. If we differentiate the equation (24) and consider the evolution in the

superhorizon scale, then we have

$$\begin{aligned} \epsilon'' + \left(2\frac{B'}{B} + \frac{H'}{H}B + \frac{H''}{H'} - 3\frac{H'}{H} - 1\right)\epsilon' + \left(-2\frac{B'^2}{B^2} + 2\frac{H'}{H}B' - \left[5\frac{H''}{H'} + 4\frac{H'}{H} + 9\right]\frac{B'}{B} \right. \\ \left. + \frac{1}{2}\frac{H'^2}{H^2}B^2 + \left[2\frac{H''}{H'} - 4\frac{H'^2}{H^2} + \frac{3}{2}\frac{H'}{H}\right]B + Q' + \left[-2\frac{H''}{H'} - \frac{H'}{H} + 1\right]Q + 4\frac{H'^2}{H^2} + 6\frac{H'}{H} + 7 \right. \\ \left. - 4\frac{Q}{B}\right)\epsilon = \frac{1}{B}F(\Psi, \Phi, Hq), \end{aligned} \quad (27)$$

where we use equations (19) and (21) and $F(\Psi, \Phi)$ is the source function for the deviation ϵ and define Q as

$$Q = \frac{H''}{H'} + \frac{H'}{H} + 3. \quad (28)$$

The above equation is different from that of the metric formalism⁸. The stability of ϵ depends on the sign of the coefficient of the term proportional to ϵ . In the metric formalism ϵ is stable as long as $B > 0$. However, the stability is complicate and need to be checked for each model in the Palatini formalism.

4.1. A particular example : $f(\hat{R}) = \beta\hat{R}^n$

We demonstrate the general consideration of the previous subsection with a specific choice for the nonlinear Lagrangian, $f(\hat{R}) = \beta\hat{R}^n$, where $n \neq 0, 2, 3$. The background is simply described by a constant effective equation of state in this model. The Hubble parameter scales as $H^2 \sim a^{-3/n}$. Then it is easy to write it with its derivatives in terms of $\ln a$

$$\frac{H'}{H} = -\frac{3}{2n}, \quad \frac{H''}{H} = \left(-\frac{3}{2n}\right)^2. \quad (29)$$

Here the scalar curvature is $\hat{R} = 3(3-n)H^2/(2n)$. From this fact, we can also find the derivatives of F with respect to $\ln a$

$$\frac{F'}{F} = \frac{F''}{F'} = \frac{3(1-n)}{n}, \quad \frac{F''}{F} = \frac{F'''}{F'} = \left(\frac{3(1-n)}{n}\right)^2. \quad (30)$$

If we use above equations (29) and (30) into (24), then we find that the deviation from the superhorizon metric evolution is null, $\epsilon = 0$.

5. Evolutions of Metric and Matter Density

5.1. Superhorizon evolution

We consider the metric evolution in superhorizon sized, $k/(aH) \ll 1$. In this case, the anisotropy relation of the equation (23) becomes

$$\Phi - \Psi \simeq (B + A)H'q, \quad (31)$$

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where A is given by

$$A = -\frac{B\left(2B\frac{H'}{H} + 5\right)}{\left(\frac{B'}{B} + \frac{3}{2}\frac{H'}{H}B + \frac{H''}{H'} + 4\right)}. \quad (32)$$

From the above equations we can find the superhorizon evolution equation of Φ and δ

$$\begin{aligned} \Phi'' + \left(\frac{B'}{B} + 2\frac{H'}{H}B + \frac{H''}{H'} - \frac{H'}{H} + 4 - C\right)\Phi + \left(\frac{B'}{B} + \frac{H'}{H}B + \frac{H''}{H'} + 3 - C\right)\Phi &\simeq 0, \\ \left(1 + \frac{B}{B+A}\right)\delta'' + \left(2\frac{B'}{B} + \frac{H'}{H}B + 2\frac{B'A - BA'}{(B+A)^2} - \frac{H'}{H}\frac{B}{B+A} + 2\frac{H''}{H'} - \frac{H'}{H} + 3\right)\delta' & \\ + \left(\frac{H'}{H}B' - 2\frac{B'^2}{B^2} - \left[4\frac{H''}{H'} + 2\frac{H'}{H} + 4\right]\frac{B'}{B} + \frac{H'^2}{H^2}B^2 + \left[\frac{H''}{H} - \frac{H'^2}{H^2} + 5\frac{H'}{H}\right]B \right. & \\ \frac{B''A - BA''}{(B+A)^2} - 2\frac{(B'A - BA')}{(B+A)^2}\frac{(B' + A')}{(B+A)} - \frac{H'}{H}\frac{B'A - BA'}{(B+A)^2} - \left(\frac{H'}{H}\right)' \frac{B}{B+A} & \\ \left. + \left[-3\frac{H''}{H} - 4\frac{H''}{H'} - 2\frac{H''^2}{H'^2} + \frac{H'^2}{H^2} - 2\frac{H'}{H} + 6\right] - 4\left[\frac{H''}{H'} + \frac{H'}{H} + 3\right]\frac{1}{B}\right)\delta = 0, \quad (33) \end{aligned}$$

where C is defined as

$$C = \frac{1}{B+A+1} \left[\frac{B'}{B} + 2\frac{H'}{H}B + 2\frac{H''}{H'} - \frac{H'}{H} + 3 \right]. \quad (34)$$

5.2. Superhorizon evolution in a particular example

Now we can check the evolution equations in the previous subsection in a particular case, $f(\hat{R}) \sim \hat{R}^n$. In this case, we can simplify the following quantities

$$B = 2(n-1), \quad A = -4(n-1) = -2B, \quad C = \frac{3}{2n} = -\frac{H'}{H}. \quad (35)$$

From this, we can also simplify the evolution equations (33) and (33)

$$\Phi'' + \frac{9-4n}{2n}\Phi' = 0, \quad (36)$$

$$\delta' = 0. \quad (37)$$

The Newtonian potential $\Phi = \text{constant}$ is a solution to the equation. Also the matter density fluctuation has the same for as general relativity, $\delta = \text{constant}$.

5.3. Subhorizon evolution

For subhorizon scales where $k/aH \gg 1$, we can find the Poisson equation from the equation (17)

$$k^2(\Phi + \Psi) \simeq -\frac{\kappa^2 a^2 \rho}{F} \delta. \quad (38)$$

If we use equations (25) and (38), then we have

$$3\Psi \simeq \left(-3\frac{\kappa^2\rho}{FH^2}\frac{a^2H^2}{k^2} + \frac{F'}{F} \right) \frac{\delta}{2} \simeq \frac{F'}{F} \frac{\delta}{2} \simeq -3\Phi. \quad (39)$$

We can find the evolution equations of Φ and δ

$$\begin{aligned} \Phi'' + \left(\frac{B'}{B} + \frac{5}{2}\frac{H'}{H}B + \frac{H''}{H'} + \frac{H'}{H} + 6 \right) \Phi' - \left(\frac{B'^2}{B^2} + \left[2\frac{H''}{H'} - 1 \right] \frac{B'}{B} - 2\frac{H'}{H}B' \right. \\ \left. - \frac{9}{4}\frac{H'^2}{H^2}B^2 + \left[2\frac{H''}{H} - 4\frac{H''}{H'} - \frac{21}{2}\frac{H'}{H} \right] B + \left[\frac{H''^2}{H'^2} - \frac{H''}{H'} + 10\frac{H'}{H} + 6 \right] \right. \\ \left. + \left[2\frac{H''}{H'} + 2\frac{H'}{H} + 6 \right] \frac{1}{B} \right) \Phi \simeq 0. \end{aligned} \quad (40)$$

$$\begin{aligned} \delta'' + \left(2\frac{B'}{B} + \frac{3}{2}\frac{H'}{H}B + 2\frac{H''}{H'} - \frac{H'}{H} + 3 \right) \delta' + \left(\frac{3}{2}\frac{H'}{H}B' - 2\frac{B'^2}{B^2} - \left[4\frac{H''}{H'} + 2\frac{H'}{H} + 4 \right] \frac{B'}{B} \right. \\ \left. + \frac{H'^2}{H^2}B^2 + \left[\frac{3}{2}\frac{H''}{H} - \frac{3}{2}\frac{H'^2}{H^2} + 5\frac{H'}{H} \right] B + \left[-3\frac{H''}{H} - 4\frac{H''}{H'} - 2\frac{H'^2}{H'^2} + \frac{H'^2}{H^2} - 2\frac{H'}{H} + 6 \right] \right. \\ \left. - 4\left[\frac{H''}{H'} + \frac{H'}{H} + 3 \right] \frac{1}{B} \right) \delta = 0. \end{aligned} \quad (41)$$

5.4. Subhorizon evolution in a particular example

We can use the previous relation (35) into the evolution equations (40) and (41)

$$\Phi'' + \frac{3(3-n)}{2n}\Phi' + \frac{3(14n^2+19n-36)}{4n^2}\Phi = 0 \quad (42)$$

$$\delta'' + \frac{3(2-n)}{2n}\delta' = 0 \quad (43)$$

The subhorizon scale evolutions of Φ and δ show different behaviors from those of general relativity as expected¹¹.

6. Conclusions

We investigate the different modified gravity models as an effective dark energy models. We can parameterize the effective equation of state of dark energy in terms of modified gravities. We show the evolution of energy density fluctuation compared to that of general relativity.

We have analyzed the stability of metric fluctuations by checking the cosmological evolution of linear perturbations in Palatini $f(R)$ gravity. We have also considered the matter density fluctuation in the Newtonian gauge.

We have shown that the stability of metric fluctuations in the high redshift limit of high curvature is not simply expressed. We need to check each model for the stability. However, we have found that the deviation from the superhorizon

metric evolution is null for a specific choice of the nonlinear Einstein-Hilbert action, $f(\hat{R}) \sim \hat{R}^n$ and stability of this model is guaranteed.

We have investigated the evolution equations of Newtonian potential and matter density contrast in super and sub-horizon scales. In the specific model, superhorizon evolutions of Newtonian potential and matter density fluctuation are same to those of general relativity. However, subhorizon evolutions show the different behaviors from the general relativity case. This will give us the method to probe the possibility of $f(R)$ theory.

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References

References

1. A. Conley et al. (The Supernova Cosmology Project), *Astrophys. J.* **644**, 1 (2006) [astro-ph/0602411]; A. G. Riess et al. (High-z Supernova Search Team), *Astrophys. J.* **607**, 665 (2004) [astro-ph/0402512].
2. D. Huterer and M. S. Turner, *Phys. Rev. D* **60**, 081301 (1999) [astro-ph/9808133].
3. C. H. Brans and R. H. Dicke, *Phys. Rev.* **124**, 925 (1961).
4. G. R. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett. B* **485**, 208 (2000) [arXiv:hep-th/0005016].
5. H. A. Buchdahl, *Mon. Not. Roy. Astron. Soc.* **150**, 1 (1970).
6. M. Ferraris, M. Francaviglia, and C. Reina, *Gen. Rel. Grav.* **14**, 243 (1982).
7. E. V. Linder and A. Jenkins, *MNRAS* **346**, 573 (2003) [astro-ph/0305286].
8. Y.-S. Song, W. Hu, and I. Sawicki, *Phys. Rev. D* **75**, 04404 (2007) [astro-ph/0610532];
9. T. Koivisto and H. Kurki-Suonio, *Class. Quant. Grav.* **23**, 2355 (2006) [astro-ph/0509422]; B. Li and M.-C. Chu, *Phys. Rev. D* **74**, 104010 (2006) [astro-ph/0610486]; K. Uddin, J. E. Lidsey, and R. Tavakol, *Class. Quant. Grav.* **24**, 3951 (2007) [arXiv:0705.0232].
10. S Lee [arXiv:0710.2395]
11. P. Zhang, *Phys. Rev. D* **73**, 123504 (2006) [astro-ph/0511218]